**CAMBRIDGE INTERNATIONAL EXAMINATIONS**

**Cambridge Ordinary Level**

# MARK SCHEME for the May/June 2015 series

# 4037 ADDITIONAL MATHEMATICS

**4037/12** Paper 1, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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**Abbreviations**

| | |
|---|---|
| awrt | answers which round to |
| cao | correct answer only |
| dep | dependent |
| FT | follow through after error |
| isw | ignore subsequent working |
| oe | or equivalent |
| rot | rounded or truncated |
| SC | Special Case |
| soi | seen or implied |
| www | without wrong working |

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|---|---|---|---|
| **1** | $k^2-4(2k+5)\quad(<0)$<br>$k^2-8k-20\quad(<0)$ | M1 | use of $b^2-4ac$, (not as part of quadratic formula unless isolated at a later stage) with correct values for $a$, $b$ and $c$<br>Do not need to see < at this point |
| | $(k-10)(k+2)\quad(<0)$ | M1 | attempt to obtain critical values |
| | critical values of 10 and –2 | A1 | correct critical values |
| | $-2<k<10$ | A1 | correct range |
| | **Alternative 1:**<br>$\frac{\mathrm{d}y}{\mathrm{d}x}=2(2k+5)x+k$ | M1 | attempt to differentiate, equate to zero and substitute $x$ value back in to obtain a $y$ value |
| | When $\frac{\mathrm{d}y}{\mathrm{d}x}=0,\ x=\frac{-k}{2(2k+5)},\ y=\frac{8k+20-k^2}{4(2k+5)}$ | M1 | consider $y=0$ in order to obtain critical values |
| | When $y=0$, obtain critical values of 10 and –2 | A1 | correct critical values |
| | $-2<k<10$ | A1 | correct range |
| | **Alternative 2:**<br>$y=(2k+5)\left(\left(x+\frac{k}{2(2k+5)}\right)^2-\frac{k^2}{4(2k+5)}\right)+1$ | M1 | attempt to complete the square and consider $'1-\frac{k^2}{4(2k+5)}'$ |
| | Looking at $1-\frac{k^2}{4(2k+5)}=0$ leads to | M1 | attempt to solve above = to 0, to obtain critical values |
| | critical values of 10 and –2 | A1 | correct critical values |
| | $-2<k<10$ | A1 | correct range |

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| 2 | $\dfrac{\tan\theta+\cot\theta}{\operatorname{cosec}\theta}=\dfrac{\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}}$ | M1 | for $\tan\theta=\dfrac{\sin\theta}{\cos\theta}$, $\cot\theta=\dfrac{\cos\theta}{\sin\theta}$ and $\operatorname{cosec}\theta=\dfrac{1}{\sin\theta}$; allow when used |
| | $=\dfrac{\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}}{\frac{1}{\sin\theta}}$ | M1 | dealing correctly with fractions in the numerator; allow when seen |
| | $=\dfrac{1}{\cos\theta}$ | M1 | use of the appropriate identity; allow when seen |
| | $=\sec\theta$ | A1 | must be convinced it is from completely correct work ( beware missing brackets) |
| | **Alternative:** | | |
| | $\dfrac{\tan\theta+\cot\theta}{\operatorname{cosec}\theta}=\dfrac{\frac{\tan^2\theta+1}{\tan\theta}}{\operatorname{cosec}\theta}$ | M1 | for either $\tan\theta=\dfrac{1}{\cot\theta}$ or $\cot\theta=\dfrac{1}{\tan\theta}$ and $\operatorname{cosec}\theta=\dfrac{1}{\sin\theta}$; allow when used |
| | $=\dfrac{\sec^2\theta}{\tan\theta\,\frac{1}{\sin\theta}}$ | M1 | dealing correctly with fractions in numerator; allow when seen |
| | $=\dfrac{\sec^2\theta}{\sec\theta}$ | M1 | use of the appropriate identity; allow when seen |
| | $=\sec\theta$ | A1 | must be convinced it is from completely correct work |
| 3 | $\mathbf{A}^{-1}=\dfrac{1}{2}\begin{pmatrix}3 & -2\\ -5 & 4\end{pmatrix}$ | B1<br>B1 | $\dfrac{1}{2}$ multiplied by a matrix<br>for matrix |
| | $\begin{pmatrix}x\\ y\end{pmatrix}=\dfrac{1}{2}\begin{pmatrix}3 & -2\\ -5 & 4\end{pmatrix}\begin{pmatrix}8\\ 9\end{pmatrix}$ | M1 | attempt to use the inverse matrix, must be pre-multiplication |
| | $\begin{pmatrix}x\\ y\end{pmatrix}=\dfrac{1}{2}\begin{pmatrix}6\\ -4\end{pmatrix}$ | | |
| | $x=3,\ y=-2$ | A1, A1 | |

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| **4** **(i)** | Area = $\left(\frac{1}{2}\times 12^2\times 1.7\right)+\left(\frac{1}{2}\times 12^2\sin(2\pi-1.7-2.4)\right)$ | B1,B1<br><br>M1 | B1 for sector area, allow unsimplified<br>B1 for correct angle *BOC*, allow unsimplified<br>correct attempt at area of triangle, allow unsimplified using *their* angle *BOC*<br>(Their angle *BOC* must not be 1.7 or 2.4) |
| | = awrt 181 | A1 | |
| **(ii)** | $BC^2=12^2+12^2-(2\times 12\times 12\cos 2.1832)$<br>or $BC=2\times 12\times\sin\left(\frac{2\pi-4.1}{2}\right)$ | M1 | correct attempt at *BC*, may be seen in **(i)**, allow if used in **(ii)**. Allow use of *their* angle *BOC*. |
| | $BC=21.296$ | A1 | |
| | Perimeter = $(12\times 1.7)+12+12+21.296$ | B1<br>M1 | for arc length, allow unsimplified<br>for a correct 'plan'<br>(an arc + 2 radii and *BC*) |
| | = 65.7 | A1 | |
| **5** **(a)** **(i)** | 20160 | B1 | |
| **(ii)** | $3\times {}^6P_4\times 2$<br>= 2160 | B1,B1 | B1 for ${}^6P_4$ (must be seen in a product)<br>B1 for all correct, with no further working |
| **(iii)** | $5\times 2\times {}^6P_4$<br>= 3600 | B1,B1<br>B1 | B1 for ${}^6P_4$ (must be seen in a product)<br>B1 for 5 (must be in a product)<br>B1 for all correct, with no further working |
| | **Alternative 1:**<br>${}^6C_4\times 5!\times 2$<br>= 3600 | B2<br>B1 | for ${}^6C_4\times 5!$<br>for ${}^6C_4\times 5!\ \times 2$ |
| | **Alternative 2:**<br>$\left({}^7P_5-{}^6P_5\right)\times 2$<br>= 3600 | B2<br>B1 | for $\left({}^7P_5-{}^6P_5\right)$<br>for $\left({}^7P_5-{}^6P_5\right)\times 2$ |
| | **Alternative 3:**<br>$2!\left({}^6P_4+\left({}^6P_1\times{}^5P_3\right)+\left({}^6P_2\times{}^4P_2\right)+\left({}^6P_3\times{}^3P_1\right)+{}^6P_4\right)$<br>= 3600 | B2<br>B1 | 4 terms correct or omission of 2! in each term<br>all correct |

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| **(b) (i)** | ${}^{14}C_4 \times {}^{10}C_4$ or ${}^{14}C_8 \times {}^{8}C_4$<br>(or numerical or factorial equivalent)<br>$= 210210$ | B1,B1 | B1 for either ${}^{14}C_4$ or ${}^{14}C_8$ as part of a product<br>B1 for correct answer, with no further working |
| **(ii)** | ${}^{8}C_4 \times {}^{6}C_4$<br>$= 1050$ | B1,B1 | B1 for either ${}^{8}C_4$ or ${}^{6}C_4$ as part of a product<br>B1 for correct answer with no further working |
| **6 (i)** | 10ln4 or 13.9 or better | B1 | |
| **(ii)** | $\left(\frac{\mathrm{d}x}{\mathrm{d}t}=\right)\frac{20t}{t^2+4}-4$ | M1 | attempt to differentiate and equate to zero |
| | | B1 | $\frac{20t}{t^2+4}$ or equivalent seen |
| | When $\frac{\mathrm{d}x}{\mathrm{d}t}=0, \frac{20t}{t^2+4}=4$ | DM1 | attempt to solve *their* $\frac{\mathrm{d}x}{\mathrm{d}t}=0$, must be a 2 or 3 term quadratic equation with real roots |
| | leading to $t^2-5t+4=0$<br>$t=1, t=4$ | A1 | for both |

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| (iii) | If $(v=)\ \dfrac{20t}{t^2+4}-4$ | | |
| | $(a=)\ \dfrac{20(t^2+4)-20t(2t)}{(t^2+4)^2}$ | M1 | attempt to differentiate *their* $\dfrac{dx}{dt}$ |
| | | A1 | $20(t^2+4)$ |
| | | A1 | $20t(2t)$ |
| | $20(4-t^2)$ or $80-20t^2$ or $4-t^2$ or equivalent expression involving $-t^2$ | A1 | $20(4-t^2)$ or $80-20t^2$ or $4-t^2$ |
| | When acceleration is 0, $t=2$ only | B1 | $t=2$, dependent on obtaining first and second A marks |
| | **Alternative 1 for first 3 marks:**<br>If$(v=)\ \dfrac{20t-4t^2-16}{t^2+4}$ | M1 | attempt to differentiate *their* $\dfrac{dx}{dt}$ |
| | $(a=)\dfrac{(t^2+4)\ (20-8t)-(20t-4t^2-16)\ (2t)}{(t^2+4)^2}$ | A1 | for $(t^2+4)(20-8t)$ |
| | | A1 | for $(20t-4t^2-16)(2t)$ |
| | **Alternative 2 for M1 mark:**<br>If $(v=)\ 20t(t^2+4)^{-1}-4$<br>$(a=)20t\left(-2t(t^2+4)^{-2}\right)+20(t^2+4)^{-1}$ | M1 | attempt to differentiate *their* $\dfrac{dx}{dt}$ |
| | **Alternative 3 for the first 3 marks**<br>If $(v=)\ (20t-4t^2-16)(t^2+4)^{-1}$<br>$(a=)(20t-4t^2-16)\left(-2t(t^2+4)^{-2}\right)+(20-8t)(t^2+4)^{-1}$ | M1 | attempt to differentiate *their* $\dfrac{dx}{dt}$ |
| | Numerator $=-2t(20t-4t^2-16)+(20-8t)\ (t^2+4)$ | A1 | for $2t(20t-4t^2-15)$ |
| | | A1 | for $(20-8t)(t^2+4)$ |
| 7 (i) | $\overrightarrow{DA}=3\mathbf{a}-\mathbf{b}$ | B1 | mark final answer, allow unsimplified |
| (ii) | $\overrightarrow{DB}=7\mathbf{a}-\mathbf{b}$ | B1 | mark final answer, allow unsimplified |
| (iii) | $\overrightarrow{AX}=\lambda(4\mathbf{a}+\mathbf{b})$ | B1 | mark final answer, allow unsimplified |
| (iv) | $\overrightarrow{DX}=3\mathbf{a}-\mathbf{b}+\lambda(4\mathbf{a}+\mathbf{b})$ | M1 | *their* **(i)** + *their* **(iii)** or equivalent valid method or $3\mathbf{a}-\mathbf{b}$ + *their* **(iii)** |
| | | A1 | Allow unsimplified |

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| **(v)** | $3\mathbf{a}-\mathbf{b}+\lambda(4\mathbf{a}+\mathbf{b})=\mu(7\mathbf{a}-\mathbf{b})$<br>Equate like vectors: $3+4\lambda=7\mu$<br>$-1+\lambda=-\mu$<br>leads to $\lambda=\frac{4}{11}$, $\mu=\frac{7}{11}$ | M1<br>DM1<br><br><br>A1,A1 | equating *their* **(iv)** and $\mu\times$ *their* **(ii)**<br>for an attempt to equate like vectors and attempt to solve 2 linear equations for $\lambda$ and $\mu$<br>A1 for each |
| **8 (i)** | $5\mathrm{e}^{2x}-\frac{1}{2}\mathrm{e}^{-2k}\quad(+c)$ | B1, B1 | B1 for each term, allow unsimplified |
| **(ii)** | $\left(5\mathrm{e}^{2k}-\frac{1}{2}\mathrm{e}^{-2k}\right)-\left(5\mathrm{e}^{-2k}-\frac{1}{2}\mathrm{e}^{2k}\right)$ | M1<br><br><br>A1 | use of limits provided integration has taken place. Signs must be correct if brackets are not included.<br>allow any correct form |
| **(iii)** | $\left(5\mathrm{e}^{2k}-\frac{1}{2}\mathrm{e}^{-2k}\right)\left(5\mathrm{e}^{-2k}-\frac{1}{2}\mathrm{e}^{2k}\right)=-60$<br>or<br>$\frac{11}{2}\mathrm{e}^{2k}-\frac{11}{2}\mathrm{e}^{-2k}=-60$<br>or equivalent<br>leading to $11e^{2k}-11e^{-2k}+120=0$ | B1<br><br><br><br>DB1 | correct expression from **(ii)** either simplified or unsimplified equated to – 60 , must be first line seen.<br><br>must be convinced as **AG** |
| **(iv)** | $11y^2+120y-11=0$<br>$(11y-1)(y+11)=0$<br>leading to<br>$k=\frac{1}{2}\ln\frac{1}{11}$, $\ln\frac{1}{\sqrt{11}}$, $-\ln\sqrt{11}$ , $-\frac{1}{2}\ln 11$ | M1<br><br><br>DM1<br>A1 | attempt to obtain a quadratic equation in $y$ or $\mathrm{e}^{2k}$ and solve to get $y$ or $\mathrm{e}^{2k}$ ( only need positive solution)<br>attempt to deal with e to get $k$ =.<br>any of given answers only. |

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| **9** | $\frac{dy}{dx} = 4 - 6\sin 2x$ | M1,A1 | M1 for attempt to differentiate<br>A1 for all correct |
| | When $x = \frac{\pi}{4}$, $y = \pi$ | B1 | for $y$ |
| | $\frac{dy}{dx} = -2$ so gradient of normal = $\frac{1}{2}$ | DM1 | for substitution of $x = \frac{\pi}{4}$ into *their* $\frac{dy}{dx}$ and use of '$m_1 m_2 = -1$', dependent on first M1 |
| | Normal equation $y - \pi = \frac{1}{2}\left(x - \frac{\pi}{4}\right)$ | DM1 | correct attempt to obtain the equation of the normal, dependent on previous DM mark |
| | When $x = 0$, $y = \frac{7\pi}{8}$ | A1 | must be terms of $\pi$ |
| | When $y = 0, x = -\frac{7\pi}{4}$ | A1 | must be terms of $\pi$ |
| | Area = $\frac{1}{2} \times \frac{7\pi}{4} \times \frac{7\pi}{8} \quad = \frac{49\pi^2}{64}$ | B1ft | Follow through on *their* $x$ and $y$ intercepts; must be exact values |
| **10 (a)** | $\cos^2 3x = \frac{1}{2}$, $\quad \cos 3x = (\pm)\frac{1}{\sqrt{2}}$<br>$3x = 45°, 135°, 225°, 315°$<br>$x = 15°, 45°, 75°, 105°$ | M1<br><br>A1,A1 | complete correct method, dealing with sec and 3, correctly<br>A1 for each correct pair |
| **(b)** | $3(\cot^2 y + 1) + 5\cot y - 5 = 0$<br>Leading to<br>$3\cot^2 y + 5\cot y - 2 = 0$ or<br>$2\tan^2 y - 5\tan y - 3 = 0$ | M1 | use of a correct identity to get an equation in terms of one trig ratio only |
| | $(3\cot y - 1)(\cot y + 2) = 0$ or<br>$(\tan y - 3)(2\tan y + 1) = 0$ | M1 | for $\cot y = \frac{1}{\tan y}$ to obtain either a quadratic equation in $\tan y$ or solutions in terms of $\tan y$; allow where appropriate |
| | $\tan y = 3$, $\quad \tan y = \frac{1}{2}$ | M1 | for solution of a quadratic equation in terms of either $\tan y$ or $\cot y$ |
| | $y = 71.6°, 251.6° \quad 153.4°, 333.4°$ | A1,A1 | A1 for each correct 'pair' |
| **(c)** | $\sin\left(z + \frac{\pi}{3}\right) = \frac{1}{2}$ | M1 | completely correct method of solution |
| | $z + \frac{\pi}{3} = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}$ | A1 | one correct solution in range |
| | $z = \frac{\pi}{2}, \frac{11\pi}{6}$ | M1 | correct attempt to obtain a second solution within the range |
| | (allow $1.57$, $5.76$) | A1 | second correct solution (and no other) |